Purpose: Understanding Coordinates and Coordinate Conversions

Instrument : TPS & GPS 1200

System 1200 makes the passing of data between the TPS1200 and GPS1200 instruments simple and seamless.

For example, a typical topographic survey may mean using GPS1200 to survey points suitable for GPS surveying and then put the CF card into the TPS1200 to complete the survey. Or control points are surveyed with GPS1200 and then maybe these points are used as backsight points for a resection when using TPS1200.

But this of course is only possible with the conversion of the coordinates of the surveyed points. Since the mixing of TPS and GPS data is becoming more and more commonplace and will continue to do so in the future, it is worth to go "back to basics" and explain the fundamentals of coordinate conversions.

1. Storage of Points within System 1200.

The first and most important thing to remember with regards to the storing of points measured using the individual TPS1200 and GPS1200 instruments is the following:

Points measured with a **TPS1200** instrument are always stored with **Local Grid** coordinates in the DBX database. Points measured with a **GPS1200** instrument are always stored with **WGS84 Geodetic** coordinates in the DBX database.

Note, the two terms local grid and WGS84 geodetic should be considered as **coordinate types.** It is only possible to convert between coordinate types if a **coordinate system** is being used. As described later, a coordinate system (typically) consists of a transformation, a local ellipsoid and a projection and should be thought of as the mathematical algorithms which allow the conversion between the two coordinate types.

Note, even during the conversion of the coordinates to other coordinate types, the original measured TPS points **always remain** stored as local grid within the DBX and the measured GPS points **always remain** stored as WGS84 geodetic within the DBX. The mathematical algorithms allow the other coordinate types to be computed and viewed, but the original stored coordinate remain the same.

2. The "Classical" 3D Method of Converting Coordinates.

FROM GPS TO TPS…

It is, of course, possible to convert coordinates in both directions - from TPS (local grid) to GPS (WGS84 geodetic) and in the other direction.

Below it is described how coordinates are converted from GPS to TPS since ultimately, local most surveyors ultimately need their surveyed points to be exported with grid coordinates.

a. STEP 1: WGS84 GEODETIC TO WGS84 CARTESIAN

As mentioned, a point surveyed with GPS1200 in the field is stored in the DBX as a WGS84 geodetic co-ordinate. As with any geodetic coordinate, it is described in terms of **Latitude** (**j**, **Longitude**(**l**) and **Height** (**h**) above the ellipsoid (in this case the WGS84 ellipsoid). To make the understanding of the coordinate conversion process a little simpler, we will consider how a point "evolves" as it goes through each stage of the coordinate conversion process to get to local grid. Our point **P** was surveyed with GPS1200 and has been stored in the DBX with the following WGS84 co-ordinates:

WGS84 latitude: 48°N WGS84 longitude: 10°E WGS84 ellipsoidal height: 500m

Knowing the **Ellipsoid** and using standard algorithms it is possible to compute the corresponding **Cartesian** co-ordinates for this same point (described in terms of **X**, **Y** and **Z**). Applying these algorithms to our point results in the following Cartesian co-ordinates:

WGS84 X: 4211089.525m WGS84 Y: 742528.701m WGS84 Z: 4717247.902m

Notice how much easier it is to imagine on the Earth where a point with geodetic co-ordinates is than Cartesian co-ordinates. Would you have known that the co-ordinates 4211089.525m, 742528.701m, 4717247.902m relate to a point on the ground near Heerbrugg in Switzerland?

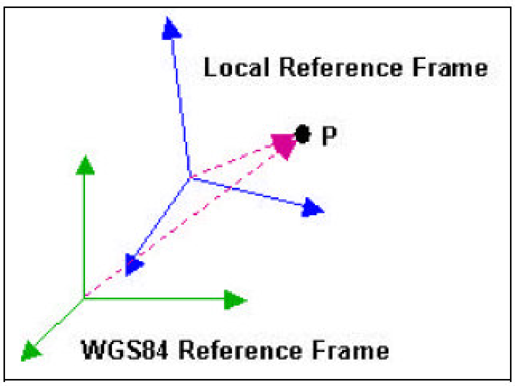
So now it is clear we need an **ellipsoid** to convert between geodetic and Cartesian coordinate and back again. However to get to local grid co-ordinates we firstly need to get to **Local Cartesian** coordinates.

b. STEP 2: WGS84 CARTESIAN TO LOCAL CARTESIAN

To get from WGS84 Cartesian to local Cartesian coordinates a **transformation** is required. A transformation consists of up to **7 Parameters**. A full 7 Parameter Transformation consists of 3 **shifts** (dX, dY and dZ), **3 rotations** (Rx, Ry, Rz) and a **scale factor**. These parameters may already be known, or may need to be computed. In some cases, not all of these parameters are required With the appropriate transformation parameters it is possible to use standard algorithms to convert between WGS84 Cartesian and local Cartesian co-ordinates.

But what really is the difference between local Cartesian co-ordinates and WGS84 Cartesian co-ordinates? They both describe the location of the same physical point in Cartesian coordinates, so why are the numbers different? It is because the origin, and/or the orientation of the two reference frames (the WGS84 and the local reference frames) are different. The transformation parameters actually mathematically describe the differences in the reference frames.

The diagram below shows two different reference frames with different origins and orientations. In our example we will use a simple **3 Parameter Transformation** where **dX=100**, **dY=-200** and **dZ=300**. This results in local Cartesian coordinates of:



Local X: 4211189.525m Local Y: 742328.701m Local Z: 4717547.902m

c. STEP 3: LOCAL CARTESIAN TO LOCAL GEODETIC

As described earlier, an ellipsoid is needed to convert between Cartesian and geodetic coordinates. Because we are now converting "on the local side" – that is, converting coordinates between local Cartesian and local geodetic, this can be called the "**Local Ellipsoid**". In this example we will use the **Bessel** ellipsoid. Using exactly the same algorithms as in Step 1 (but with a different ellipsoid) we can compute local geodetic co-ordinates. In our example we obtain the local geodetic coordinates:

Local latitude: 48°00' 0.82316"N Local longitude: 9°59'49.66165"E
Local ellipsoidal ht: 1468.783m

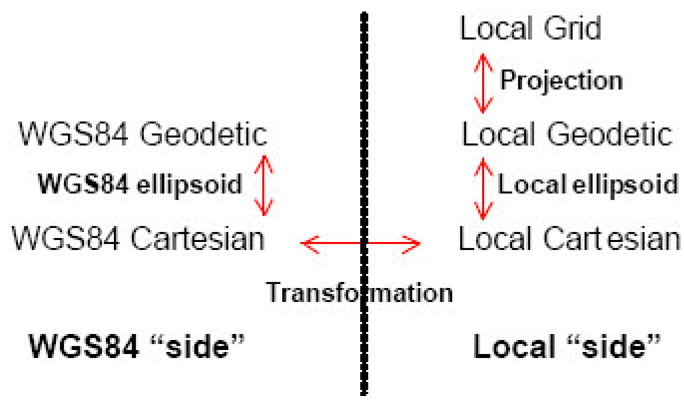
d. STEP 4: LOCAL GEODETIC TO LOCAL GRID

The final step in the coordinate conversion process is to compute grid coordinates – to do this we need a **Projection**. Hundreds of projections exist, all of them are used to convert **Geodetic** co-ordinates (on a curved surface) to **Grid** co-ordinates (on a plane surface). As for the local ellipsoid, the projection which is used for a specific country is the one which fits best to the shape of the country with the minimum of distortion. For example a Transverse Mercator projection is suitable for a country, which is long and thin in a north-south direction whereas a Lambert projection may better suit a more "square" shaped country. Some larger countries or States within the US will even use more than one projection to cover the country.

In our example we use the **Universal Transverse Mercator 32** and standard algorithms to obtain the following grid co-ordinates:

Easting: 790830.175m Northing: 319665.347m Ellipsoidal ht: 1486.783m

Now we have completed the conversion of the co-ordinates of our point from WGS84 geodetic co-ordinates to local grid with the following coordinate types:



REMEMBER

*Most important to remember is that points measured with a **TPS1200** instrument are always stored with **Local Grid** coordinates and points measured with a **GPS1200** instrument are always stored with **WGS84 Geodetic** coordinates in the DBX database. This never changes – using a coordinate system simply allows the coordinates to be converted, but the points themselves are not "re-stored".*

*Generally speaking, a coordinate system consists of several "elements"– the **Transformation**, the **Local Ellipsoid** and the **Projection** and it is these individual elements which allow the coordinate conversions to be made.*

Within LGO and System1200, this combination of transformation, local ellipsoid and projection is known as a **Classic 3D** coordinate system. It is clear that the local ellipsoid and projection must be known in order to use a Classic 3D coordinate system. If the transformation is not known and must be determined, then it is necessary that all common points used to determine the transformation must be known in position and height. But what if you do not know the local ellipsoid and projection? Maybe you need to use GPS to survey an area which uses a completely arbitrary coordinate system - is it still possible to use GPS to complete this survey? Or what if the common points being used to determine the transformation are not known in position and height, but only position or height? In these cases, it would be necessary to compute and use a **OneStep** transformation. Regardless of which transformation type is used, the basic idea of matching common points – that is, matching the control points on the ground for which the grid coordinates are known and which have been measured with GPS - is the same.

3. THE ONESTEP COORDINATE SYSTEM

Imagine you want to use GPS to survey a quarry. For years, the quarry has been measured with TPS where the origin of this “grid system” is a survey marker in the corner of a field with the coordinates 1000, 5000 with additional control points positioned around the quarry. These control points have been here for some years, they fit together “pretty good” and are of sufficient accuracy for regular surveys of the quarry. Clearly, in order to use GPS to survey this quarry we need GPS to fit into this grid system and give the same coordinates for points as if they had been measured with TPS. The Classical 3D approach cannot be used here – there is no local ellipsoid and projection – simply an arbitrary grid! The **OneStep** transformation is ideal for this situation. Remember, the ultimate goal of all coordinate systems is to convert coordinates between WGS84 geodetic and grid coordinates. In order to do this with the OneStep coordinate system, the **position** and **height** components of this transformation are treated separately.

a. HOW IT WORKS

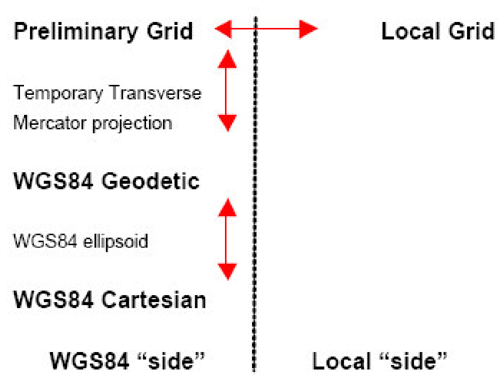
POSITION COMPONENT

Imagine you have now measured all the control points with GPS in the area in which you wish to work and therefore know the coordinates of the common points in both WGS84 geodetic coordinates (the GPS measured points) and local grid coordinates (the easting and northing of the control points). It is now possible to match these common points and compute the position component.

Note, when matching points in order to compute the OneStep transformation it is possible to match points by position only – so even if the height of a control point is not known, the control point can still be used, with only the easting and northing of the point being used. The position component of the OneStep coordinate system can be thought of as being computed in 2 steps (these 2 steps are “invisible” to the user).

The first step is that the WGS84 geodetic coordinates of the points are converted to grid coordinates using a “temporary” Transverse Mercator projection. The central meridian of this projection passes through the centre of gravity of the common points. This results in **preliminary grid co-ordinates** for the GPS measured points. These preliminary grid coordinates are never seen on System1200 or in LGO (they are of no interest to a user). The second step is to match these **preliminarygrid co-ordinates** with the **local grid control points** and compute the “best-fitting” **easting** and **northing shifts**, **rotation** and **scale factor** between these two sets of points. The positional component of the transformation is now computed. What this means is that the GPS coordinates are “squeezed” to fit into the local grid coordinates.

Note, it is possible to compute the positional component with only one common point being matched. In this case, the rotation is zero (grid north will point in the same direction as WGS84 north) and the scale is 1. The conversion process is now modified as shown below.



Note that different to the Classic 3D transformation, there are no local geodetic or local Cartesian computed. So now we have the position component of the OneStep transformation (this is actually nothing more than a 2D Helmert transformation).

HEIGHT COMPONENT
Now we need to compute the height component of the OneStep transformation. Again, the common points have been matched and so the height component can be computed. Similar to the way that points could be matched in position only, it is also possible to match points by height only. It is even possible to compute a OneStep transformation without knowing the height of any of the control points (in this case the height of the computed local points have the same height as the WGS84 coordinates).
If only one point is matched in height, then the WGS84 heights are simply shifted to fit to thatone local height control point.

If **two** or **three** points are matched in height then a plane is fitted to these points. If **three** or more points are matched in height, a best fitting tilted plane is computed to approximate the local heights.

b. WHAT DOES ALL THIS REALLY MEAN?
So now you know the theory of the OneStep transformation, but what really happens when you then attach this coordinate system to a job and survey the quarry with GPS? Basically GPS measured coordinates now "fit" to the real world – where the real world in this case is the existing quarry grid system.

In our quarry, the control points of course do not fit perfectly together – this can be seen when examining the residuals of the matched points during the determination of the coordinate system and is graphically shown below for position.

It could be said that the accuracy of GPS is actually "too good" for the quarry! But the aim was to retain the original TPS grid system (including any errors) such that when any point is now measured with GPS it would give the same coordinates as if it had been measured with TPS. This is now the case.

So the OneStep transformation sounds wonderful! It can be used when the local ellipsoid and projection is not known and can be used with control points where only the positions or heights of the points are known. Why is it not used everywhere for every survey?

There is of course a limit. The main disadvantage of the OneStep transformation is that it is limited to areas of about 10km square. This is because the WGS84 geodetic coordinates are projected to the "preliminary" grid co-ordinates using a Transverse Mercator projection with a scale of 1 with the central meridian passing through the centre of gravity of the common points. It is extremely unlikely that the control points in the grid system were also originally surveyed using the same scale factor which results from the OneStep Transverse Mercator. Differences will therefore quickly grow the further you are away from the centre of the common points. How big can the errors grow if the area is extended?

This is very hard to answer and mainly depends on how quickly (also if) the scale factor of the local points change within the area. Errors may sometimes quickly reach several centimetres. This question will be looked at in more detail in a future newsletter.

REMEMBER
A OneStep transformation can be computed even when the local ellipsoid or projection is not known. Common points can be matched in position and height, in position only or even in height only. The OneStep transformation treats the heights and position components of points separately. There is a limit over the area in which a OneStep transformation can be used – this is due to the distortions resulting from scale errors. The errors increase depending on the distance from the centre of the common points.

4. THE FINAL TRANSFORMATION TYPE – THE TWOSTEP

This covers the theory of the **TwoStep** transformation.

QUICK RECAP

THE CLASSIC 3D TRANSFORMATION
The advantage of the **Classical 3D** transformation is that it is the most rigorous transformation type - it is a similarity transformation, which keeps the full geometrical information.

The disadvantage is that that knowledge of the **local ellipsoid** and the **map projection** is required and all common points have to be known in **position and height**.

THE ONESTEP TRANSFORMATION
The advantage of the OneStep coordinate system is that it is not necessary to know the local ellipsoid and projection, which makes it ideal to use in areas which use a completely height.

The disadvantage of the OneStep transformation is that it is limited to areas of about 10km square.arbitrary coordinate system. Additionally, it is possible to use common points for which the coordinates are known in only position or

THE TWOSTEP TRANSFORMATION
The **TwoStep** transformation combines the advantages of the two approaches. It allows common points to be used which are known only in position or height, but is not restricted to smaller areas. However, as for the Classic 3D transformation, it is necessary to know the local ellipsoid and map projection.

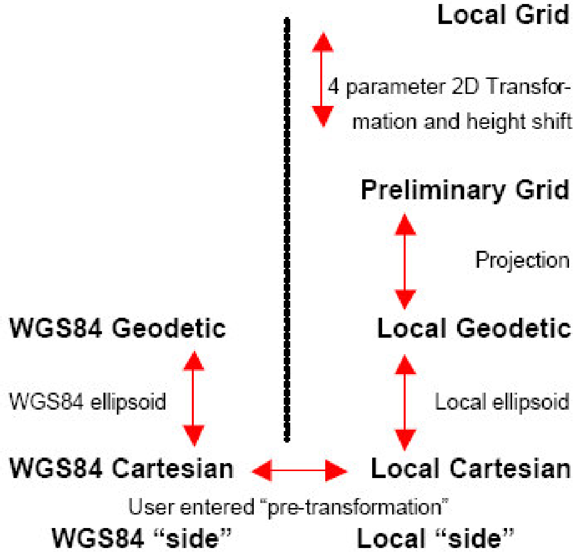
a. HOW DOES THE TWOSTEP WORK?
As the name would suggest, there are two steps in converting coordinates from WGS84 to local coordinates. In the first step the **WGS84 Cartesian** coordinates are shifted closely to the local datum using a given Classical 3D **Pre- Transformation** to give **local Cartesian** coordinates. (This pre-transformation is entered by the user - see later).

The **local Cartesian** coordinates are then converted to **local geodetic** coordinates using the known local ellipsoid and then converted to **preliminary grid**, but unlike the OneStep transformation which uses an arbitrary Transverse Mercator projection, the TwoStep transformation uses the true map projection on which the local points are based. So far this is basically the same as converting coordinates from WGS84 to local grid. So far we have converted our GPS measured points to local grid – we can now match these grid coordinates to the known grid coordinates of the local control points – this is the second step of the TwoStep coordinate system.

The two sets of grid coordinates are matched in exactly the same way as with the OneStep transformation. The final part of the transformation is therefore a **2D positional transformation** and **height shift**. The full TwoStep coordinate conversion process is shown in the following diagram.

b. WHY USE A TWOSTEP?
This probably sounds all very complicated, sowhy use a TwoStep transformation?

Compared to the OneStep transformation, the first step of the TwoStep transformation (when the WGS84 coordinates are converted to the preliminary grid) avoids any distortions due to the fact that the preliminary grid co-ordinates are built on a different ellipsoid to the local points. Even more importantly the influence of the scale factor of the map projection is now taken into account before the final 2D transformation is done. For these reasons the transformation will fit much better over larger areas than a OneStep transformation. The height part is independent of the position transformation and is identical to the approach taken for the OneStep.



Note, when using a TwoStep transformation it is not possible to see the "preliminary grid" coordinates either in LGO or on the System1200 instruments. Only the final local grid coordinates are shown. This is correct since the "preliminary grid" coordinates have no practical use.

c. WHICH PRE-TRANSFORMATION TO USE?
So far, we have only briefly mentioned the pretransformation – as shown in the diagram above, this is the 7 parameter transformation which is used to convert from WGS84 Cartesian coordinates to local Cartesian coordinates. The actual parameters may be known and can then simply be entered as a transformation on both System1200 instruments and LGO.

However, even if transformation parameters are not known it is still possible to use the TwoStep transformation. Create and select a"null" transformation (zero shifts, rotations and scale) – this then gives you the advantage from the benefits of using the TwoStep transformation!

REMEMBER
*The main advantage of the TwoStep transformation is that unlike the OneStep transformation, it is not limited to smaller areas and common points can be matched in **position and height**, in **position only** or **height only** to compute the transformation. It is necessary to know the local ellipsoid and projection and also necessary to select a pretransformation to use (although this may be a null transformation*).